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DETERMINATION OF THE THERMAL CHARACTERISTICS OF A TWO-DIMENSIONAL LAMINAR BOUNDARY LAYER IN A COMPRESSIBLE GAS WITHOUT A LONGITUDINAL PRESSURE GRADIENT, IN THE VICINITY OF THE FORWARD STAGNATION POINT

I. I. Suksov

Translation of "Ob opredelenii teplovykh kharakteristik dvukhmernogo laminarnogo pogranichnogo sloya v szhimayemom gaze pri otsutstvii prodol'nogo gradiyenta davleniya i v okrestnosti peredney kriticheskoy tochki".
Izvestiya Vysshikh Uchebnykh Zavedeniy, Seriya Aviatsionnaya Tekhnika, No.1, pp.27-37, 1961.

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I.I.Suksov

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Simple approximate solutions in finite form are derived for the determination of friction, recovery, and heat transfer coefficients for a plane plate in the vicinity of the forward stagnation point, applicable to the cases of high aerodynamic heating in the two-dimensional boundary layer of a compressible gas without pressure gradient. The solutions are extended to the boundary layers of bodies of revolution, over the Stepanov-Mangler transformation.

Current theoretical works on the boundary layer strive to reflect as fully as possible the effect of high aerodynamic heating on friction and heat transfer. For this purpose, more accurate relationships are used for the viscosity coefficient, under consideration of the variability in specific heat and Prandtl number as well as dissociation and ionization of gas. Considerable attention is devoted to an investigation of the dynamic and thermal boundary layers in the absence of a pressure gradient (in particular on a plane plate) and in the region of the forward stagnation point. In the case of a laminar boundary layer, the problem often reduces to a system of ordinary differential equations (self-mapping or similar solutions) which is sometimes written in integral form (Bibl.2, 8, 9). The equations are solved numerically, usually by the method of

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* Numbers in the margin indicate pagination in the original foreign text.

successive approximations.

Along with these solutions it is useful to develop approximate methods which, in addition to satisfactory accuracy, possess great simplicity and yield results in a finite form. This would facilitate a study of the effect of various independent parameters in a sufficiently wide range of variation.

In this work, simple approximate solutions are obtained in finite form, yielding adequate results in determinations of friction, recovery, and heat-transfer coefficients for the case of a plane plate in the vicinity of the forward stagnation point.

It may be assumed that analogous solutions can be derived when taking into account the effects associated with high aerodynamic heating.

1. Approximate Linearization of Boundary-Layer Equations

Using the conventional formula for the viscosity coefficient,

$$\frac{\mu}{\mu_{0\delta}} = K \frac{T}{T_{0\delta}}, \tag{1.1}$$

the equation of a two-dimensional steady laminar boundary layer in Dorodnitsyn's (Bibl.5) independent variables /28

$$\xi = \int_0^x \frac{p}{p_{0\delta}} dx, \qquad \eta = \int_0^y \frac{\rho}{\rho_{0\delta}} dy, \tag{1.2}$$

will take the form

$$u \frac{\partial u}{\partial \xi} + w \frac{\partial u}{\partial \eta} = \frac{\tau}{1 - u_\delta^2} u_\delta u_\delta' + \nu_{0\delta} K \frac{\partial^2 u}{\partial \eta^2}, \tag{1.3}$$

$$\frac{\partial u}{\partial \xi} + \frac{\partial w}{\partial \eta} = 0, \tag{1.4}$$

$$u \frac{\partial \theta}{\partial \xi} + w \frac{\partial \theta}{\partial \eta} = \frac{\nu_{0\delta} K}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} + \nu_{0\delta} K \left(1 - \frac{1}{Pr}\right) \frac{\partial^2 u^2}{\partial \eta^2}, \tag{1.5}$$

where

x and y = coordinates from which the first is read off along the surface from the forward stagnation point and the second along the outward normal to the surface,

u and v = corresponding velocity components,

$$w = \frac{v}{\tau} + \frac{p_{*\delta}}{p} u \frac{\partial \eta}{\partial x},$$

p, ρ, T = pressure, density, and temperature respectively,

$\tau = \frac{T}{T_{*\delta}}$, $\theta = \frac{T}{T_{*\delta}}$, $T_* = T + \frac{u^2}{2Ic_p}$ = stagnation temperature,

$\alpha^2 = \frac{u^2}{2Ic_p T_{*\delta}}$,

μ = viscosity coefficient, $\nu = \frac{\mu}{\rho}$, $K = (1 + \bar{C})(\theta_w + \bar{C})^{-1}\sqrt{\theta_w}$, $\bar{C} =$ $= \frac{C}{T_{*\delta}}$,

C = a constant (for air, $C = 114\,^\circ K$),

I = mechanical equivalent of heat,

c_p = specific heat at constant pressure,

$Pr = \frac{\mu c_p}{k}$ = Prandtl number,

k = coefficient of thermal conductivity,

$\mu'_\delta = \frac{du_\delta}{d\xi}$.

Indexes:

$*$ = stagnation,

δ = outer limit of the boundary layer,

w = wall.

In deriving eqs.(1.3) - (1.5) and hereafter we will use also the equation of state of an ideal gas:

$$p = R\rho T \tag{1.6}$$

(which holds also for the stagnation parameters) and the Bernoulli equation:

$$\frac{p}{p_{*\delta}} = (1 - \alpha_\delta^2)^{\frac{\varkappa}{\varkappa - 1}}, \tag{1.7}$$

where

$$x = \frac{c_p}{c_v}, \quad c_v = \text{specific heat at constant volume.}$$

The quantities c_p, c_v, and Pr are considered to be constant. The Prandtl number is assumed to be close to unity.

Let us examine the solution of eqs.(1.3) - (1.5) for a plane plate and in the vicinity of the forward stagnation point, using a representation of a boundary layer of finite thickness and assuming the thicknesses of the dynamic and thermal boundary layers to be identical. The boundary conditions will be /29

$$u = w = 0, \quad \theta = \theta_w \quad \text{or} \quad \frac{\partial\theta}{\partial\eta} = 0, \quad \text{if} \quad \eta = 0,$$

$$u = u_\delta(\xi), \quad \frac{\partial u}{\partial\eta} = 0, \quad \theta = 1, \quad \frac{\partial\theta}{\partial\eta} = 0, \quad \text{if} \quad \eta = \Delta(\xi). \tag{1.8}$$

Here, $\Delta(\xi)$ is the arbitrary thickness of the boundary layer in variables [eq.(1.2)]. Let us introduce the variables:

$$\varphi(\bar{\eta}) = \frac{u}{u_\delta}, \quad \bar{\eta} = \frac{\eta}{\Delta}. \tag{1.9}$$

Let us approximately express all terms of eqs.(1.3) and (1.5), except terms with $\frac{\partial^2 u}{\partial\eta^2}$ and $\frac{\partial^2\theta}{\partial\eta^2}$, by means of the velocity and stagnation temperature profiles, whose form is known beforehand:

$$\frac{u}{u_\delta} = \bar{\varphi}(\bar{\eta}), \quad \frac{T_0}{T_{0\delta}} = \bar{\theta}(\bar{\eta}, \theta_w, a_\delta^2, \mathrm{Pr}). \tag{1.10}$$

We will express w by integrating the continuity equation (1.4) provided that $w|_{\bar{\eta}=0} = 0$. Furthermore, we will use the identity

$$\int \bar{\eta}\bar{\varphi}' d\bar{\eta} = \bar{\eta}\bar{\varphi}' - \int \bar{\varphi} d\bar{\eta},$$

and reduce eqs.(1.3), (1.5), and the boundary conditions (1.8) to the form

$$\frac{\partial^2\varphi}{\partial\bar{\eta}^2} = -\frac{u_\delta'\zeta(\bar{\theta} - \bar{\varphi}^2)}{1 - a_\delta^2} - (u_\delta'\zeta + 0.5u_\delta\zeta')\bar{\varphi}'\int_0^{\bar{\eta}} \bar{\varphi} d\bar{\eta}, \tag{1.11}$$

$$\frac{\partial^2\theta}{\partial\bar{\eta}^2} = -\Pr(u_\delta'\zeta + 0{,}5u_\delta\zeta')\frac{\partial\bar{\theta}}{\partial\bar{\eta}}\int_0^{\bar{\eta}}\bar{\varphi}\,d\bar{\eta} + (1-\Pr)\,\alpha_\delta^2\cdot(\varphi^2)'' +$$

$$+\Pr u_\delta\zeta\frac{\partial\theta_w}{\partial\xi}\cdot\bar{\varphi}\frac{\partial\bar{\theta}}{\partial\theta_w} + 2\Pr\alpha_\delta^2 u_\delta'\zeta\frac{\partial\bar{\theta}}{\partial\alpha_\delta^2}\cdot\bar{\varphi}, \tag{1.12}$$

$$\text{at}\quad \eta=0,\quad \varphi=0,\quad \theta=\theta_w \quad \text{or} \quad \frac{\partial\theta}{\partial\bar{\eta}}=0,$$

$$\text{at}\quad \bar{\eta}=1,\quad \varphi=1,\quad \varphi'=0,\quad \theta=1,\quad \frac{\partial\theta}{\partial\bar{\eta}}=0. \tag{1.13}$$

Here,

$$\zeta=\frac{\Delta^2}{K\nu_{e\delta}},\quad \zeta'=\frac{d\zeta}{d\xi},\quad \bar{\varphi}'=\frac{d\bar{\varphi}}{d\bar{\eta}},\quad \bar{\varphi}''=\frac{d^2\bar{\varphi}}{d\bar{\eta}^2}.$$

Equations (1.11) and (1.12) are linear with respect to the unknown functions φ and θ and are readily integrated in a finite form if we prescribe the functions $\bar{\varphi}$ and $\bar{\theta}$ as polynomials of $\bar{\eta}$.

The two additive functions that appear on integration are determined by the conditions: $\varphi'(1) = 0$ and $\varphi(0) = 1$. Satisfaction of still another necessary condition $\varphi(1) = 1$ furnishes an ordinary differential equation for ζ. /30

The approximating expressions (1.10) must satisfy all conditions (1.13) for φ and θ. Furthermore, their selection is subject to additional conditions resulting from eqs.(1.11), (1.12) since these represent the conditions of vanishing of the derivatives of eqs.(1.10) at $\bar{\eta} = 1$.

In the presence of heat transfer, the equation for ζ will have the form

$$u_\delta\zeta' + \left[s + \frac{s_1\alpha_\delta^2}{1-\alpha_\delta^2} - \frac{s_2(1-\theta_w)}{1-\alpha_\delta^2}\right]u_\delta'\zeta = d, \tag{1.14}$$

where

s, s_1, s_2, d = constants dependent on the selection of the functions [eq.(1.10)].

2. The Heat-Insulated Plane Plate

In this case, u_δ = const, $u_\delta' = 0$.

The boundary condition for the dimensionless temperature at the wall is

$$\left.\frac{\partial\bar{\theta}}{\partial\bar{\eta}}\right|_{\bar{\eta}=0}=0. \tag{2.1}$$

The expressions (1.10) will take the form

$$\bar{\varphi}=0.5(3\bar{\eta}-\bar{\eta}^3), \tag{2.2}$$

$$\bar{\theta}=1-0.75\,(1-\mathrm{Pr})\,\alpha_\delta^2\,(1-3\bar{\eta}^2+2\bar{\eta}^3). \tag{2.3}$$

The profile (2.2) was used elsewhere (Bibl.7) in generalizing the approximate solution (Bibl.1) for a compressible gas at Pr = 1 and in the absence of heat transfer between the gas and the body. It satisfies the boundary conditions (1.13) and also the condition $\bar{\varphi}''(0) = 0$ resulting from eq.(1.11). The profile (2.3) obeys the boundary conditions (1.13) [the condition (2.1) is taken at the wall], and also the condition

$$\left.\frac{\partial^2\bar{\theta}}{\partial\bar{\eta}^2}\right|_{\bar{\eta}=0}=4.5\,(1-\mathrm{Pr})\,\alpha_\delta^2\,, \tag{2.4}$$

which follows from eq.(1.12) with the use of eq.(2.2).

Equation (1.11), with consideration of eq.(2.2), takes the form

$$\frac{\partial^2\varphi}{\partial\bar{\eta}^2}=-\frac{3u_\delta\,\zeta'}{32}\,(6\bar{\eta}^2-7\bar{\eta}^4+\bar{\eta}^6).$$

Solving this equation with the boundary conditions (1.13) for φ, we find the velocity profile

$$\varphi=\frac{u}{u_\delta}=1.621\bar{\eta}-1.091\bar{\eta}^4+0.509\bar{\eta}^6-0.039\bar{\eta}^8 \tag{2.5}$$

and the equation for determining ζ

$$u_\delta\,\zeta'=d, \tag{2.6}$$

where d = 23.27.

Integrating eq.(1.6) in the case of $\zeta(0) = 0$ and returning to the /31 variable x, we obtain

$$\zeta = \frac{d}{u_\delta}(1-a_\delta^2)^{\frac{\varkappa}{\varkappa-1}} x. \tag{2.7}$$

For the local coefficient of friction $c_f = \frac{2\tau_w}{\rho_\delta u_\delta^2}$ we obtain, as before (Bibl.7), the expression

$$\sqrt{Re_x}\, c_f = 0.672, \tag{2.8}$$

where $Re_x = \frac{u_\delta x}{\nu_\delta}$. In deriving eq.(2.8), we used the relations

$$\tau_w = \mu_w \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

eqs.(2.5), (2.7), and also the relations

$$\rho_\delta = \rho_{0\delta}(1-a_\delta^2)^{\frac{1}{\varkappa-1}}, \quad K\nu_{0\delta} = \nu_\delta(1-a_\delta^2)^{\frac{2-\varkappa}{\varkappa-1}},$$

which follow from eqs.(1.1), (1.6), (1.7).

The proposed solution agrees well with the self-mapping solution (Bibl.6) which, on the right-hand side of eq.(2.8), gives a value of 0.664.

Equation (1.12) in this case takes the form

$$\frac{\partial^2\theta}{\partial\bar{\eta}^2} = (1-\Pr)\, a_\delta^2 (\varphi^2)'' - \frac{\Pr\cdot(1-\Pr)}{16} a_\delta^2 u_\delta \zeta' (27\bar{\eta}^3 - 27\bar{\eta}^4 - 4{,}5\bar{\eta}^5 + 4{,}5\bar{\eta}^6).$$

Integrating it for the cases $\left.\frac{\partial\theta}{\partial\bar{\eta}}\right|_{\bar{\eta}=0} = 0$, $\theta|_{\bar{\eta}=1} = 1$ and using eq.(2.6), we find the stagnation temperature profile

$$\theta = 1 - (1-\Pr)\, a_\delta^2 [P_1(\eta) - \Pr\cdot P_2(\bar{\eta})], \tag{2.9}$$

where

$$P_1(\bar{\eta}) = 1 - 2.25\bar{\eta}^2 + 1.5\bar{\eta}^4 - 0.25\bar{\eta}^6,$$
$$P_2(\bar{\eta}) = 0.616 - 1.963\bar{\eta}^5 + 1.309\bar{\eta}^6 + 0.156\bar{\eta}^7 - 0.117\bar{\eta}^8.$$

Setting $\eta = 0$ in eq.(2.9), we find the dimensionless temperature of the surface

$$\theta_{act.} = \frac{T_{act.}}{T_{0\delta}} = 1 - (1 - Pr)\,(1 - 0.616\,Pr)\,\alpha_\delta^2, \qquad (2.10)$$

and the recovery coefficient

$$r = \frac{T_{act.} - T_\delta}{T_{0\delta} - T_\delta} = 1 - (1 - Pr)\,(1 - 0.616\,Pr). \qquad (2.11)$$

For air (Pr = 0.72), we obtain r = 0.844.

Figure 1 gives a comparison of the values of r for a plane plate calculated by eq.(2.11) (curve 1) with the results of the self-mapping solution (Bibl.6) (curve 2) and with the data of the calculation by the known formula $r = \sqrt{Pr}$ (curve 3). Satisfactory coincidence is obtained in the range of Pr = 0.65 - 1.

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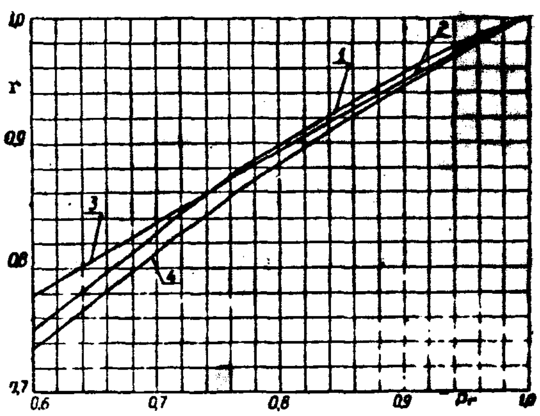

Fig.1

3. Heat Transfer on a Plane Plate

In this case, we will take the condition for the wall temperature

$$\theta|_{\bar\eta=0} = \theta_w. \qquad (3.1)$$

For an approximate solution of the problem, let us use the velocity profile (2.2) and the stagnation temperature profile

$$\bar\theta = 1 - 0.5\,(1 - \theta_w)\,(2 - 3\bar\eta + \bar\eta^3) - 1.125\,(1 - Pr)\,\alpha_\delta^2\,(\bar\eta - 2\bar\eta^2 + \bar\eta^3), \qquad (3.2)$$

found with respect to the conditions (1.13) for θ [the condition (3.1) is taken on the wall] and to the condition (2.4).

Equations (2.5) - (2.8) remain valid here.

Limiting ourselves to the case of the absence of a longitudinal temperature gradient, we can represent eq.(1.12) in the form

$$\frac{\partial^2\theta}{\partial\bar{\eta}^2} = -\frac{\Pr u_\delta \zeta'}{2}[(1-\theta_w)(1.125\bar{\eta}^2 - 1.312\bar{\eta}^4 + 0.188\bar{\eta}^6) + $$
$$+ (1-\Pr)\alpha_\delta^2(0.844\bar{\eta}^2 - 3.375\bar{\eta}^3 + 2.391\bar{\eta}^4 + 0.562\bar{\eta}^5 - 0.422\bar{\eta}^6).$$

Integrating the equation written for the conditions $\theta|_{\bar{\eta}=0} = \theta_w$, $\theta|_{\bar{\eta}=0} = 1$, we obtain

$$\left.\begin{aligned} \theta = \theta_w + (1-\theta_w)\bar{\eta} - (1-\Pr)\alpha_\delta^2(\bar{\eta} - \bar{\eta}^3) + 0.5\Pr\cdot d[(1-\theta_w)Q_1(\bar{\eta}) - \\ - (1-\Pr)\alpha_\delta^2 Q_2(\bar{\eta})], \\ Q_1(\bar{\eta}) = 0.053\bar{\eta} - 0.094\bar{\eta}^4 + 0.044\bar{\eta}^6 - 0.003\bar{\eta}^8, \\ Q_2(\bar{\eta}) = 0.013\bar{\eta} + 0.070\bar{\eta}^4 - 0.169\bar{\eta}^5 + 0.080\bar{\eta}^6 + 0.013\bar{\eta}^7 - 0.008\bar{\eta}^8. \end{aligned}\right| \quad (3.3)$$

Proceeding from a determination of the local Nusselt number

$$\mathrm{Nu}_x = \frac{xq}{k_\delta(T_{\text{act.}} - T_w)}$$

(q is the thermal flux) and using eq.(2.10) as well as the expressions

$$\left.\begin{aligned} q = k_w\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\frac{k_w T_{0\delta}(1-\alpha_\delta^2)^{\frac{x}{x-1}}}{\theta_w\cdot\Delta}\Phi(\theta_w, \alpha_\delta^2, \Pr), \\ \Phi(\theta_w, \alpha_\delta^2, \Pr) = \left(\frac{\partial\theta}{\partial\bar{\eta}}\right)_{\bar{\eta}=0} = (1+0.622\Pr)(1-\theta_w) - \\ - (1+0.150\Pr)(1-\Pr)\alpha_\delta^2, \end{aligned}\right|$$ /33

$\frac{k_w}{k_\delta} = \frac{\theta_w}{1-\alpha_\delta^2}$ [the latter is obtained because of the constancy of c_p, Pr, and eqs.(1.1)], we finally obtain

$$\frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}} = 0.2071(1+0.622\Pr)A_1, \quad (3.4)$$

where

$$a_1 = \frac{1 - \theta_w - a_2(1 - \mathrm{Pr})\,\sigma_1^2}{1 - \theta_w - (1 - \mathrm{Pr})(1 - 0.616\,\mathrm{Pr})},$$

$$a_2 = \frac{1 + 0.150\,\mathrm{Pr}}{1 + 0.622\,\mathrm{Pr}}$$

At Pr = 1, a_1 = 1. Assuming $a_1 \approx 1$ (in view of the proximity of Pr to 1), we write in place of eq.(3.4):

$$\frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}} = 0.336 f(\mathrm{Pr}), \tag{3.5}$$

where

$$f(\mathrm{Pr}) = \frac{1 + 0.622\,\mathrm{Pr}}{1.622}.$$

The value 0.336f (Pr) for Pr = 0.6 - 1.0 differs little from the value of $0.332\sqrt[3]{\mathrm{Pr}}$ used in the theory of heat transfer (Bibl.10). The values of these quantities are shown in Fig.2.

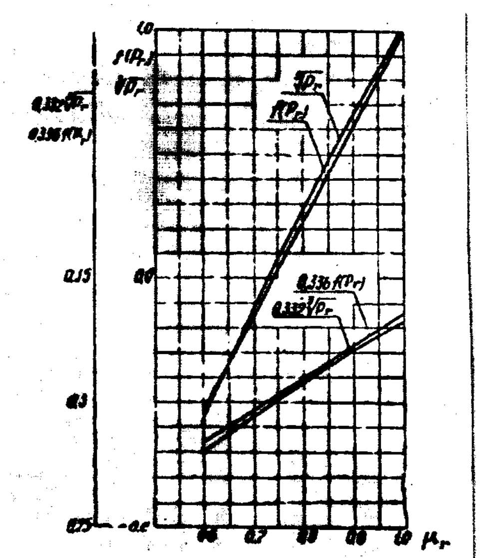

Fig.2

On the basis of eqs.(2.8) and (3.5) we obtain

$$\frac{c_f \mathrm{Re}_x}{\mathrm{Nu}_x} = \frac{2}{f(\mathrm{Pr})}.$$

Figure 2 shows that, in the range of Prandtl numbers under consideration, the function f(Pr) differs little from $\sqrt[3]{Pr}$, i.e., for the given solution the Reynolds analogy between friction and heat transfer is well satisfied.

When using the local heat-transfer coefficient $\alpha_x = \frac{k_\delta Nu_x}{x}$ the specific thermal flux will be expressed by the formula

$$q = \alpha_x (T_{act.} - T_w).$$

The coefficient α_x is expressed in terms of the flow parameters and the Stanton number h_x:

$$\alpha_x = 3600 g c_p \rho_\delta u_\delta h_x,$$

where

$$h_x = \frac{1}{Pr} \cdot \frac{Nu_x}{Re_x}.$$

Taking into account (3.5) we obtain /34

$$h_x = \frac{0{,}460 f(Pr)}{Pr \sqrt{Re_x}}.$$

For air, considering Pr = 0.72, we have

$$h_x = \frac{0{,}410}{\sqrt{Re_x}}.$$

In the known formulas for h_x, a value of 0.408 is substituted for 0.410.

Let us note the following:

Applying the condition (2.1) to the profile (3.3) will yield

$$\theta_{act.} = 1 - \frac{(1 - Pr)(1 - 0{,}150\,Pr)\,\alpha_\delta^2}{1 + 0622\,Pr}. \quad (3.6)$$

The corresponding recovery coefficient is

$$r = 1 - \frac{(1 - Pr)(1 - 0{,}150\,Pr)}{1 + 0{,}622\,Pr}. \quad (3.7)$$

If, in eq.(3.4) the expression (3.6) is used instead of eq.(2.10), then

$a_1 = 1$ regardless of θ_w, α_δ^2, and Pr.

For comparison, Fig.1 shows the results of calculating r from eq.(3.7) (curve 4). It is obvious that this formula is less accurate than eq.(2.11).

4. Heat Transfer in the Vicinity of the Forward Stagnation Point

In this case, let us set, as usual $u_\delta = \beta x$, $\alpha_\delta^2 \approx 0$ (β is a constant). Then eq.(1.14) takes the form

$$x\zeta' + [s - s_2(1-\theta_w)]\zeta = \frac{d}{\beta}.$$

Considering θ_w = const, we find the solution of this equation:

$$\zeta = \frac{\Delta^2}{K\nu_{w\delta}} = \frac{d}{\beta[s - s_2(1-\theta_w)]} = \text{const.}$$

whence $\zeta' = 0$.

The form parameter $\lambda = \dfrac{u_\delta' \zeta}{1 - \alpha_\delta^2}$ in this case is equal to

$$\lambda_0 = \frac{d}{s - s_2(1-\theta_w)} = \text{const.}$$

To solve eqs.(1.11), (1.12) let us use polynomials of the fourth degree as the functions $\bar{\varphi}$ and $\bar{\theta}$:

$$\bar{\varphi} = 4\bar{\eta} - 6\bar{\eta}^2 + 4\bar{\eta}^3 - \bar{\eta}^4, \tag{4.1}$$

$$\bar{\theta} = \theta_w + (1-\theta_w)(2\bar{\eta} - 2\bar{\eta}^3 + \bar{\eta}^4). \tag{4.2}$$

The coefficients of these polynomials are determined from the conditions /35 (1.3) and from the auxiliary conditions: $\bar{\varphi}''(1) = \bar{\varphi}'''(1) = \bar{\theta}''(1) = 0$.

In the case under consideration eqs.(1.11), (1.12) take the form

$$\frac{\partial^2 \bar{\varphi}}{\partial \bar{\eta}^2} = \lambda_0 [-\theta_w(1 - 2\bar{\eta} + 2\bar{\eta}^3 - \bar{\eta}^4) - 2\bar{\eta} + 8\bar{\eta}^2 - 14\bar{\eta}^3 + 15\bar{\eta}^4 - 11{,}2\bar{\eta}^5 + 5{,}6\bar{\eta}^6 - 1{,}6\bar{\eta}^7 + 0{,}2\bar{\eta}^8), \tag{4.3}$$

$$\frac{\partial^2 \bar{\theta}}{\partial \bar{\eta}^2} = -\Pr \cdot \lambda_0 (1-\theta_w)(4\bar{\eta}^2 - 4\bar{\eta}^3 - 10\bar{\eta}^4 + 19{,}6\bar{\eta}^5 - 14\bar{\eta}^6 + 5{,}2\bar{\eta}^7 - 0{,}8\bar{\eta}^8). \tag{4.4}$$

Integrating eq.(4.3) and using the conditions (1.13) for φ, we obtain

$$\varphi = \lambda_0 [R_1(\bar{\eta})\,\theta_w + R_2(\bar{\eta})], \tag{4.5}$$

$$\lambda_0 = \frac{1}{0.0244 + 0.0667\theta_w}, \tag{4.6}$$

where

$$R_1(\bar{\eta}) = 0.3\bar{\eta} - 0.5\bar{\eta}^2 + 0.3333\bar{\eta}^3 - 0.1\bar{\eta}^5 + 0.0333\bar{\eta}^6,$$

$$R_2(\bar{\eta}) = 0.0778\bar{\eta} - 0.3333\bar{\eta}^3 + 0.6667\bar{\eta}^4 - 0.7\bar{\eta}^5 + 0.5\bar{\eta}^6 - 0.2667\bar{\eta}^7 + 0.1\bar{\eta}^8 - 0.0222\bar{\eta}^9 + 0.0022\bar{\eta}^{10}.$$

Integrating eq.(4.4) for the conditions: $\theta|_{\bar{\eta}=w} = \theta_w$, $\theta|_{\bar{\eta}=1} = 1$ will yield

$$\theta = \theta_w + (1-\theta_w)\,[(1 + 0.08\,\mathrm{Pr}\,\lambda_0)\,\bar{\eta} - \mathrm{Pr}\,\lambda_0 R_3(\bar{\eta})], \tag{4.7}$$

where

$$R_3(\bar{\eta}) = 0.0333\bar{\eta}^4 - 0.2\bar{\eta}^5 - 0.3333\bar{\eta}^6 + 0.4667\bar{\eta}^7 - 0.25\bar{\eta}^8 + 0.0722\bar{\eta}^9 - 0.0089\bar{\eta}^{10}.$$

The specific thermal flux is

$$q = k_w\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{k_w T_{e\delta}}{\theta_w \Delta}\,\Phi(\theta_w,\ \mathrm{Pr}),$$

where

$$\Delta = \sqrt{\frac{k\nu_{e\delta}\lambda_0}{\beta}},\quad \Phi(\theta_w,\ \mathrm{Pr}) = \left(\frac{\partial\theta}{\partial\bar{\eta}}\right)_{\bar{\eta}=0} = (1-\theta_w)(1 + 0.08\,\mathrm{Pr}\,\lambda_0).$$

As usual, let us assume $k_\delta = k_{*\delta}$, $T_{act} = T_{*\delta}$ in the vicinity of the forward stagnation point. We will also consider that, as a consequence of eq.(1.1) and of the constancy of c_p and Pr, we have $\frac{k_w}{k_{*\delta}} = \theta_w$. Then, for the Nusselt number

$$\mathrm{Nu} = \frac{ql}{k_{e\delta}(T_{e\delta} - T_w)}$$

we obtain the relation

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = \frac{1 + 0.08\,\mathrm{Pr}\,\lambda_0}{\sqrt{\lambda_0}}\sqrt{K}\sqrt{\frac{\bar{\alpha}'_{\delta cr}}{\alpha_\infty}}, \tag{4.8}$$

where

$\bar{\alpha}'_{\delta\,cr} = \dfrac{\beta l}{\sqrt{2I c_p T_{*\delta}}}$ = value of the quantity $\dfrac{d\alpha_\delta}{d\bar{x}}$ in the vicinity of /36 the stagnation (critical) point,

$\bar{x} = \frac{x}{\ell}$ where ℓ is the characteristic dimension,

∞ = as subscript, conditions of the relative flow.

The heat transfer coefficient α is connected with the Nu number by the relation

$$\alpha = \frac{k_{ex}\,\mathrm{Nu}}{l}$$

We note that in eq.(4.8) the quantity $\vec{\alpha}_{\delta_{cr}}$ is half the quantity in the analogous formula by Kalikhman (Bibl.3). In essence, these formulas differ only by the coefficients; in the latter, the coefficient is $N_1 = 0.570\ \mathrm{Pr}^{0.4}$, whereas in the former it is

$$N_2 = \frac{1 + 0.08\lambda_0 \mathrm{Pr}}{\sqrt{\lambda_0}},$$

where at $\theta_w = 1$ (then, $\lambda_0 = 10.96$), it equals $0.302\ (1 + 0.877\ \mathrm{Pr})$.

We should also mention the solution of the problem of forced laminar convection from a cylinder close to the forward stagnation point (Bibl.10) which agrees well with the experimental data. In this solution, the factor $\alpha_3(\mathrm{Pr})$ coincides with N_1 at $\mathrm{Pr} = 0.6 - 1.1$.

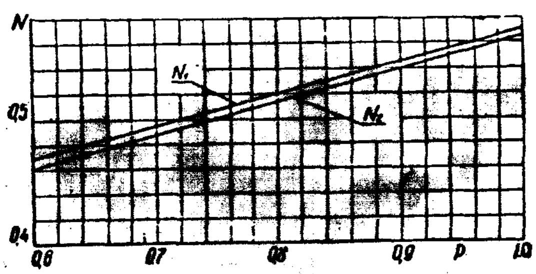

Fig.3

A comparison (Fig.3) shows the satisfactory agreement of the quantities N_1 and N_2.

By means of the well-known Stepanov-Mangler transformation, the solutions are readily extended to the boundary layer of bodies of revolution (Bibl.4, 6).

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